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One electron atoms in both a squeezed vacuum and a magnetic field via path integral methods

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Abstract. In the present paper we study the dynamics of one-electron atoms in the presence of both a linearly polarized squeezed vacuum and a magnetic field along the polarization vector of the photonic field. We adopt the dipole approximation and approach the problem *via* path integral methods. After integrating over the light variables for certain initial and final squeezed vacuum states we treat the path integral over the spatial variables *via* Monte-Carlo methods. As an application we calculate the survival probability of the ground state of a one-electron atom for various values of the magnetic field.

PACS. 03.65.Db Functional analytical methods – 31.15.Kb Path-integral methods

1 Introduction

Through time considerable effort has been made in applying path integral methods in quantum optics. Certain dynamical groups have been studied [1,2] and the propagators of particular forms of Hamiltonians describing non-classical states have been obtained exactly [3,4]. Other Hamiltonians can and have been investigated only numerically *via* methods such as Monte-Carlo.

Additionally certain non-classical states have been achieved in the laboratory. It is fifteen years that quadrature-squeezed light has been produced experimentally [5]. The interaction of non-classical light with matter appears as a challenging area of research.

On the other hand there is a vast bibliography on the effect of magnetic fields on matter and particularly atoms. Hydrogen in a magnetic field is of particular interest in classical, semi-classical and quantum dynamics [6,7].

Presently we are going to study the influence of squeezed vacuum generated by a degenerate parametric amplifier on an atomic bound state [8–10] in the presence of a magnetic field *via* a formalism developed recently by the author [11,12].

The paper proceeds in the following order. In Section 2 we describe the full Hamiltonian of an electron in the presence of a magnetic field, a non-classical field and a potential to be specified at will, we give the full propagator and we integrate over the field variables. In Section 3 and as an application we derive the survival probability of the ground state of the atom by Monte-Carlo methods. Finally in Section 4 we give our conclusions.

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2 System Hamiltonian and path integration

The full Hamiltonian H can be written as the sum of three terms: the electron Hamiltonian H_{e}, in the potential $V(\mathbf{r})$ and the magnetic field $\mathbf{H}$, the squeezed field one H_{f} and the interaction term H_{I} between the electron and the squeezed field

$$H = H_{\mathrm{e}} + H_{\mathrm{f}} + H_{\mathrm{I}}. \tag{1}$$

Particularly the electron Hamiltonian is given as

$$H_{\mathrm{e}} = \frac{1}{2}\left(\mathbf{p} - \frac{1}{c}\mathbf{A}\right)^2 + V(\mathbf{r}) \tag{2}$$

where

$$\mathbf{A} = \frac{1}{2}\mathbf{H} \times \mathbf{r}.$$

The Hamiltonian of squeezed light has the form

$$H_{\mathrm{f}}(t) = \omega(t)a^{+}a + f(t)a^2 + f^{\star}(t)a^{+2}. \tag{3}$$

In the case of production of squeezed light by a degenerate parametric amplifier the squeezed light Hamiltonian is given as

$$H_{\mathrm{f}}(t) = \omega a^{+}a + \kappa\left(\mathrm{e}^{2\mathrm{i}\omega t}a^2 + \mathrm{e}^{-2\mathrm{i}\omega t}a^{+2}\right). \tag{4}$$

Finally the interaction Hamiltonian in the length form is given as

$$H_{\mathrm{I}} = -e\mathbf{r}\cdot\mathbf{E}_{\mathrm{f}}. \tag{5}$$

The second quantized form of the field operator of squeezed light is given as:

$$\mathbf{E}_{\mathrm{f}}(\mathbf{r}) = \frac{1}{\sqrt{V}}\mathrm{i}l(\omega)\hat{\varepsilon}\left[a\mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{r}-\omega t)} - a^{+}\mathrm{e}^{-\mathrm{i}(\mathbf{k}\cdot\mathbf{r}-\omega t)}\right] \tag{6}$$

where V is the quantization volume and $l(\omega)$ is a real function of frequency given as $l(\omega) = \sqrt{\hbar\omega/2\varepsilon_0}$.

In the dipole approximation ($\mathrm{e}^{\mathbf{ik}\cdot\mathbf{r}} \approx 1$) which we adopt here, since the spatial dimension of the atom radiated is much less than the wavelength of the electromagnetic field the atom is in, the field operator can be written as

$$\mathbf{E}_\mathrm{f} = \frac{1}{\sqrt{V}} il(\omega)\hat{\varepsilon}\left(a\mathrm{e}^{-\mathrm{i}\omega t} - a^+\mathrm{e}^{\mathrm{i}\omega t}\right) \tag{7}$$

and H_I takes the form:

$$H_\mathrm{I} = -\frac{1}{\sqrt{V}}\mathrm{i}el(\omega)\hat{\varepsilon}\cdot\mathbf{r}(t)\left(a\mathrm{e}^{-\mathrm{i}\omega t} - a^+\mathrm{e}^{\mathrm{i}\omega t}\right). \tag{8}$$

Now we combine the terms (3) and (8) involving field variables in the term

$$\begin{aligned}H_0(a^+, a; t) &= H_\mathrm{f} + H_\mathrm{I}\\ &= \omega a^+a + f(t)a^2 + f^\star(t)a^{+2} + g(t)a + g^\star(t)a^+ \end{aligned}\tag{9a}$$

where

$$g(t) = -\frac{1}{\sqrt{V}}\mathrm{i}el(\omega)\hat{\varepsilon}\cdot\mathbf{r}(t)\mathrm{e}^{-\mathrm{i}\omega t}. \tag{9b}$$

The propagator corresponding to (9a) has been derived by Hillery and Zubairy. Here we use their result to obtain the full propagator corresponding to H, with the field variables appearing in (9) integrated, thus resulting in a path integral of only the spatial variables. It is given by the expression

$$K(\alpha_\mathrm{f}, \mathbf{r}_\mathrm{f}; \alpha_\mathrm{i}, \mathbf{r}_\mathrm{i}; t) = \int_{\mathbf{r}(0)=\mathbf{r}_\mathrm{i}}^{\mathbf{r}(t)=\mathbf{r}_\mathrm{f}} D\mathbf{r}\exp\begin{bmatrix}\mathrm{i}\int_0^t \mathrm{d}\tau\left[\frac{\dot{\mathbf{r}}^2(\tau)}{2} - V(\mathbf{r}(\tau)) + \frac{H}{2c}(x\dot{y} - y\dot{x})\right]\\ -\mathrm{i}\int_0^t \mathrm{d}\tau\left[2f(\tau)X(\tau) + f(\tau)Z^2(\tau) + g(\tau)Z(\tau)\right]\\ -\frac{1}{2}\left(|\alpha_\mathrm{f}|^2 + |\alpha_\mathrm{i}|^2\right) + Y(t)\alpha_\mathrm{f}^\star\alpha_\mathrm{i} + X(t)\left(\alpha_\mathrm{f}^\star\right)^2\\ -\mathrm{i}\alpha_\mathrm{i}^2\int_0^t \mathrm{d}\tau f(\tau)Y^2(\tau) + Z(t)\alpha_\mathrm{f}^\star\\ -\mathrm{i}\alpha_\mathrm{i}\int_0^t \mathrm{d}\tau\left[g(\tau) + 2f(\tau)Z(\tau)\right]Y(\tau)\end{bmatrix} \tag{10a}$$

where $X(t)$ satisfies the following Riccati differential equation

$$\frac{\mathrm{d}X}{\mathrm{d}t} = -2\mathrm{i}\omega(t)X - 4\mathrm{i}f(t)X^2 - \mathrm{i}f^\star(t) \tag{10b}$$

with initial condition $X(0) = 0$.

$Y(t)$ and $Z(t)$ are given as

$$Y(t) = \exp\left[-\mathrm{i}\int_0^t \mathrm{d}\tau\left[\omega(\tau) + 4f(\tau)X(\tau)\right]\right] \tag{10c}$$

$$\begin{aligned}Z(t) = &-\mathrm{i}\int_0^t \mathrm{d}\tau\left[g^\star(\tau) + 2g(\tau)X(\tau)\right]\\ &\times\exp\left[-\mathrm{i}\int_\tau^t \mathrm{d}\tau'\left[\omega(\tau') + 4f(\tau')X(\tau')\right]\right].\end{aligned} \tag{10d}$$

Now we consider the special case of the Hamiltonian (4), describing squeezed light generated by a degenerate parametric amplifier, by setting

$$f(t) = \kappa\mathrm{e}^{2\mathrm{i}\omega t}. \tag{11}$$

Then (9a) is written as

$$\begin{aligned}H_0(a^+, a; t) &= H_\mathrm{f} + H_\mathrm{I}\\ &= \omega a^+a + \kappa\mathrm{e}^{2\mathrm{i}\omega t}a^2 + \kappa\mathrm{e}^{-2\mathrm{i}\omega t}a^{+2} + g(t)a + g^\star(t)a^+\end{aligned} \tag{12}$$

and by using formulas (10) we obtain

$$X(t) = \frac{1}{2\mathrm{i}}\mathrm{e}^{-2\mathrm{i}\omega t}\tanh(2\kappa t) \tag{13}$$

$$Y(t) = \mathrm{e}^{-\mathrm{i}\omega t}\mathrm{sech}(2\kappa t) \tag{14}$$

$$Z(t) = \frac{1}{\sqrt{V}}el(\omega)\int_0^t \mathrm{d}\tau\,\hat{\varepsilon}\cdot\mathbf{r}(\tau)\zeta(\tau, t) \tag{15}$$

where the function $\zeta(\tau, t)$ in (15) is given as

$$\begin{aligned}\zeta(\tau, t) = &\left[\mathrm{e}^{2\mathrm{i}\omega\tau} + \mathrm{i}\mathrm{e}^{-2\mathrm{i}\omega\tau}\tanh(2\kappa\tau)\right]\\ &\times\cosh(2\kappa\tau)\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{sech}(2\kappa t).\end{aligned} \tag{16}$$

The propagator (10a) with diagonal photonic field variables can be written as

$$K(\alpha, \mathbf{r}_\mathrm{f}; \alpha, \mathbf{r}_\mathrm{i}; t) = \int_{\mathbf{r}(0)=\mathbf{r}_\mathrm{i}}^{\mathbf{r}(t)=\mathbf{r}_\mathrm{f}} D\mathbf{r}\exp\begin{bmatrix}\mathrm{i}\int_0^t \mathrm{d}\tau\left[\frac{\dot{\mathbf{r}}^2(\tau)}{2} - V(\mathbf{r}(\tau)) + \frac{H}{2c}(x\dot{y} - y\dot{x})\right]\\ -\frac{1}{2}\ln\cosh(2\kappa t) + A - B|\alpha|^2 + \frac{C}{2}\alpha^{\star 2}\\ +\frac{C_1}{2}\alpha^2 + D_1\alpha + D\alpha\end{bmatrix} \tag{17}$$

where

$$D(t) = \frac{1}{\sqrt{V}}el(\omega)\int_0^t \mathrm{d}\tau\,\hat{\varepsilon}\cdot\mathbf{r}(\tau)\zeta(\tau, t) \tag{18a}$$

$$D_1 = -\frac{1}{\sqrt{V}}el(\omega)\int_0^t \mathrm{d}\tau\,\hat{\varepsilon}\cdot\mathbf{r}(\tau)\left[Y(\tau)\mathrm{e}^{-\mathrm{i}\omega\tau} + \mathrm{i}\theta(\tau, t)\right] \tag{18b}$$

$$C_1(t) = -\mathrm{i}\tanh(2\kappa t) \tag{18c}$$

$$C(t) = -\mathrm{i}\mathrm{e}^{-2\mathrm{i}\omega t}\tanh(2\kappa t) \tag{18d}$$

$$B(t) = 1 - Y(t) \tag{18e}$$

$$\begin{aligned}A(t) = &-\frac{1}{V}e^2l^2(\omega)\int_0^t \mathrm{d}\tau\int_0^\tau \mathrm{d}\rho\,\hat{\varepsilon}\cdot\mathbf{r}(\tau)\hat{\varepsilon}\cdot\mathbf{r}(\rho)\\ &\times\left[\zeta(\rho, \tau)\mathrm{e}^{-\mathrm{i}\omega\tau} + \mathrm{i}\lambda(t, \tau, \rho)\right].\end{aligned} \tag{18f}$$

$\theta(\tau, t)$ and $\lambda(t, \tau, \rho)$ are given as

$$\begin{aligned}\theta(\tau, t) = &\left[\mathrm{e}^{2\mathrm{i}\omega\tau} + \mathrm{i}\mathrm{e}^{-2\mathrm{i}\omega\tau}\tanh(2\kappa\tau)\right]\cosh(2\kappa\tau)\\ &\times\left[\tanh(2\kappa t) - \tanh(2\kappa\tau)\right]\end{aligned} \tag{18g}$$

$$\begin{aligned}\lambda(t, \tau, \rho) = &\left[\mathrm{e}^{2\mathrm{i}\omega\tau} + \mathrm{i}\mathrm{e}^{-2\mathrm{i}\omega\tau}\tanh(2\kappa\tau)\right]\\ &\times\left[\mathrm{e}^{2\mathrm{i}\omega\rho} + \mathrm{i}\mathrm{e}^{-2\mathrm{i}\omega\rho}\tanh(2\kappa\rho)\right]\cosh(2\kappa\tau)\\ &\times\cosh(2\kappa\rho)\left[\tanh(2\kappa t) - \tanh(2\kappa\rho)\right].\end{aligned} \tag{18h}$$

Therefore we can integrate the diagonal propagator over the field variable α between a final $|0; \mu, \nu\rangle$ and an initial $|0; \mu', \nu'\rangle$ squeezed vacuum state of the field to obtain the following reduced propagator for the motion of the electron

$$\widetilde{K}(\mathbf{r}_\mathrm{f},\mathbf{r}_\mathrm{i};t)=\frac{1}{\sqrt{\mu\mu' N(t)}}\int_{\mathbf{r}(0)=\mathbf{r}_\mathrm{i}}^{\mathbf{r}(t)=\mathbf{r}_\mathrm{f}} D\mathbf{r}\exp\left\{\mathrm{i}S_\mathrm{tot}\left[\mathbf{r}\right]\right\}$$
$$=\frac{1}{\sqrt{\mu\mu' N(t)}}\widetilde{K}_0(\mathbf{r}_\mathrm{f},\mathbf{r}_\mathrm{i};t) \quad (19a)$$

where

$$S_\mathrm{tot}\left[\mathbf{r}\right]=\int_0^t\left[\frac{\dot{\mathbf{r}}^2(\rho)}{2}-V(\mathbf{r}(\rho))+\frac{H}{2c}(x\dot{y}-y\dot{x})\right]\mathrm{d}\rho$$
$$+\frac{1}{V}e^2l^2(\omega)\int_0^t\mathrm{d}\rho\int_0^\rho\mathrm{d}\sigma\,\hat{\boldsymbol{\varepsilon}}\cdot\mathbf{r}(\rho)\hat{\boldsymbol{\varepsilon}}\cdot\mathbf{r}(\sigma)\phi(t,\rho,\sigma) \quad (19b)$$

$$N(t)=$$
$$\cosh(2\kappa t)-2\mathrm{e}^{-\mathrm{i}\omega t}+\left(\cosh(2\kappa t)-\mathrm{i}\frac{\nu}{\mu}\sinh(2\kappa t)\right)\mathrm{e}^{-2\mathrm{i}\omega t}$$
$$-\mathrm{i}\frac{\nu'}{\mu'}\sinh(2\kappa t)-\frac{\nu\nu'}{\mu\mu'}\cosh(2\kappa t) \quad (19c)$$

$$\phi(t,\rho,\sigma)=\mathrm{i}\zeta(\sigma,\rho)\mathrm{e}^{-\mathrm{i}\omega\rho}-\lambda(t,\rho,\sigma)$$
$$+\frac{\mathrm{i}}{K(t)}\begin{bmatrix} B(t)\zeta(\rho,t)\left[Y(\sigma)\mathrm{e}^{-\mathrm{i}\omega\sigma}+\mathrm{i}\theta(\sigma,t)\right]\\ +B(t)\zeta(\sigma,t)\left[Y(\rho)\mathrm{e}^{-\mathrm{i}\omega\rho}+\mathrm{i}\theta(\rho,t)\right]\\ -\left[C_1(t)-(\nu/\mu)\right]\zeta(\rho,t)\zeta(\sigma,t)\\ -\left[C(t)-(\nu'/\mu')\right]\left[Y(\rho)\mathrm{e}^{-\mathrm{i}\omega\rho}+\mathrm{i}\theta(\rho,t)\right]\\ \times\left[Y(\sigma)\mathrm{e}^{-\mathrm{i}\omega\sigma}+\mathrm{i}\theta(\sigma,t)\right]\end{bmatrix} \quad (19d)$$

and

$$K(t)=1-2\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{sech}(2\kappa t)+\left(1-\mathrm{i}\frac{\nu}{\mu}\tanh(2\kappa t)\right)\mathrm{e}^{-2\mathrm{i}\omega t}$$
$$-\mathrm{i}\frac{\nu'}{\mu'}\tanh(2\kappa t)-\frac{\nu\nu'}{\mu\mu'}\cdot \quad (19e)$$

Now we perform the Markovian approximation in the Lagrangian of the above action by setting $\hat{\boldsymbol{\varepsilon}}\cdot\mathbf{r}(\sigma)$ equal to its latest value. On performing this approximation we tacitly assume that the electron's dynamic variables depend only in their present value and no memory appears in the interaction with the photonic field. The following integral arises

$$\nu(t,\rho)=\int_0^\rho\phi(t,\rho,\sigma)\mathrm{d}\sigma \quad (20)$$

and finally the action (19b) becomes

$$S_\mathrm{tot}\left[\mathbf{r}\right]=\int_0^t\left[\frac{\dot{\mathbf{r}}^2(\rho)}{2}-V(\mathbf{r}(\rho))+\frac{H}{2c}(x\dot{y}-y\dot{x})\right]\mathrm{d}\rho$$
$$+\frac{1}{V}e^2l^2(\omega)\int_0^t\mathrm{d}\rho\left(\hat{\boldsymbol{\varepsilon}}\cdot\mathbf{r}(\rho)\right)^2\nu(t,\rho). \quad (19b)$$

3 The Monte-Carlo method and application to one-electron atoms

Now we apply the above theory to the case of one-electron atoms in both a static magnetic field and squeezed vacuum.

In that case the potential in (2) is given as

$$V(\mathbf{r})=-\frac{Z}{r}\cdot \quad (21)$$

We are going to use a Monte-Carlo approach. We proceed in the following way.

In its discrete form the above path integral can be written as

$$\widetilde{K}_0(\mathbf{r}_\mathrm{f},\mathbf{r}_\mathrm{i};t)=\frac{1}{\sqrt{2\pi\mathrm{i}\varepsilon}^3}\lim_{N\to\infty}\prod_{n=1}^N\int\frac{\mathrm{d}\mathbf{r}_n}{\sqrt{2\pi\mathrm{i}\varepsilon}^3}\exp\left\{\mathrm{i}\sum_{n=1}^N S_n\right\} \quad (22)$$

where its measure in spherical coordinates can be written as

$$\frac{1}{\sqrt{2\pi\mathrm{i}\varepsilon}^3}\prod_{n=1}^N\frac{\mathrm{d}\mathbf{r}_n}{\sqrt{2\pi\mathrm{i}\varepsilon}^3}=\frac{1}{\sqrt{2\pi\mathrm{i}\varepsilon}^3}\prod_{n=1}^N\frac{\mathrm{d}r_n\,r_n^2\cos\vartheta_n\mathrm{d}\varphi_n}{\sqrt{2\pi\mathrm{i}\varepsilon}^3}\cdot \quad (23)$$

On supposing that we have directed the z-axis along the direction of the linear polarization the exponential term can be written as

$$\mathrm{e}^{\mathrm{i}S_n}=\int_{-\infty}^{\infty}\mathrm{d}w_n\,\delta(w_n-r_n\cos\vartheta_n)\mathrm{e}^{\mathrm{i}S_n^w} \quad (24a)$$

where

$$S_n^w=\varepsilon\left[\frac{1}{2\varepsilon^2}\left(\mathbf{r}_n-\mathbf{r}_{n-1}\right)^2-V(r_n)\right.$$
$$\left.+\frac{H}{2c}\left(x_n\frac{y_n-y_{n-1}}{\varepsilon}-y_n\frac{x_n-x_{n-1}}{\varepsilon}\right)+\nu_n w_n^2\right]\cdot \quad (24b)$$

By performing the transformation $\lambda_n\to(\sqrt{2\pi\omega}/\sqrt{V})\lambda_n$ we have transfer the missing factors in other terms in the expression (31) of the propagator (see below).

Additionally the kinetic energy term can be expressed as

$$\exp\left\{\frac{\mathrm{i}}{2\varepsilon}\left(\mathbf{r}_n-\mathbf{r}_{n-1}\right)^2\right\}=$$
$$\exp\left\{\frac{\mathrm{i}}{2\varepsilon}\left(r_n^2+r_{n-1}^2-2r_nr_{n-1}\cos\Delta\vartheta_n\right)\right\}. \quad (25)$$

On using the following representation of the delta function

$$\delta(w_n-r_n\cos\vartheta_n)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}\lambda_n\,\mathrm{e}^{-\mathrm{i}\lambda_n w_n}\mathrm{e}^{\mathrm{i}\lambda_n r_n\cos\vartheta_n}$$
$$=\frac{1}{2\pi}\sum_{l_n=0}^{\infty}(2l_n+1)\mathrm{i}^{l_n}P_{l_n}(\cos\vartheta_n)$$
$$\times\int_{-\infty}^{\infty}\mathrm{d}\lambda_n\,\mathrm{e}^{-\mathrm{i}\lambda_n w_n}j_{l_n}(\lambda_n r_n) \quad (26)$$

$$\widetilde{K}(\mathbf{r}_{\rm f},\mathbf{r}_{\rm i};t)=\frac{1}{\sqrt{2\pi{\rm i}\varepsilon}}\prod_{n=1}^{N}\left[\int_0^\infty\frac{{\rm d}r_n\cos\vartheta_n{\rm d}\varphi_n}{\sqrt{2\pi{\rm i}\varepsilon}}\right]\prod_{n=1}^{N+1}\left[\int_{-\infty}^{\infty}\frac{{\rm d}\lambda_n}{2\pi}\right]$$
$$\times\prod_{n=1}^{N+1}\left[\begin{array}{l}\sum_{p_n=-\infty}^{\infty}\sum_{l'_n=0}^{\infty}\sum_{l_n=0}^{\infty}\sum_{m_n=-l_n}^{l_n}\sqrt{2l'_n+1}\sqrt{4\pi}{\rm i}^{l'_n}\widetilde{J}_{l_n+\frac{1}{2}}\left(\frac{r_nr_{n-1}}{\varepsilon}\right)j_{l'_n}\left(\frac{\sqrt{2\pi\omega}\lambda_nr_n}{\sqrt{V}}\right)f_n(\lambda_n)\\ \times Y_{l'_n0}(\vartheta_n,\varphi_n)Y_{l_nm_n}(\vartheta_n,\varphi_n)\exp({\rm i}p_n(\varphi_n-\varphi_{n-1}))Y^\star_{l_nm_n}(\vartheta_{n-1},\varphi_{n-1})\\ \times J_{p_n}\left(\frac{H}{2c}r_nr_{n-1}\right)\exp\left\{{\rm i}\varepsilon\left[\frac{1}{2\varepsilon^2}(r_n-r_{n-1})^2-V(r_n)\right]\right\}\end{array}\right] \quad (31)$$

changing the order of integration between w_n and λ_n using the identity

$$\int_{-\infty}^{\infty}{\rm d}w_n\,{\rm e}^{-{\rm i}\lambda_nw_n}{\rm e}^{\varepsilon\nu_nw_n^2}=\sqrt{-\frac{\pi}{\varepsilon\nu_n}}\exp\left[\frac{\lambda_n^2}{4\varepsilon\nu_n}\right]=f_n(\lambda_n) \quad (27)$$

as well as the expansions

$${\rm e}^{{\rm i}z\cos\Delta\vartheta_n}=\sqrt{\frac{\pi}{2z}}\sum_{l=0}^{\infty}J_{l+\frac{1}{2}}(z){\rm i}^l(2l+1)P_l\left(\cos\Delta\vartheta_n\right) \quad (28)$$

$${\rm e}^{{\rm i}z\sin\varphi}=\sum_{n=-\infty}^{\infty}J_n(z){\rm e}^{{\rm i}n\varphi} \quad (29)$$

$$\frac{2l+1}{4\pi}P_l\left(\cos\Delta\vartheta_n\right)=\sum_{m=-l}^{l}Y_{lm}\left(\vartheta_n,\varphi_n\right)Y^\star_{lm}\left(\vartheta_{n-1},\varphi_{n-1}\right) \quad (30)$$

we obtain the following expression after certain manipulations

see equation (31) above

where

$$\widetilde{J}_m(z)=\sqrt{-2\pi{\rm i}z}{\rm e}^{{\rm i}z}J_m(z) \quad (32)$$

and $r_{N+1}=r_{\rm f}$, $r_0=r_{\rm i}$.

On performing the angular integrations and keeping leading terms in V we obtain the following expression

$$\widetilde{K}(\mathbf{r}_{\rm f},\mathbf{r}_{\rm i};t)=\sum_{p=-\infty}^{\infty}\sum_{l=0}^{\infty}\sum_{k=0}^{\infty}\sum_{m=-k}^{k}\frac{1}{r_{\rm f}r_{\rm i}}\widetilde{K}_{plk}(r_{\rm f},r_{\rm i};t)$$
$$\times Y_{l0}(\vartheta_b,\varphi_b)Y_{km}(\vartheta_b,\varphi_b)\exp({\rm i}p(\varphi_b-\varphi_a))Y^\star_{km}(\vartheta_a,\varphi_a). \quad (33)$$

For instance we have for $N=0$

$$\widetilde{K}_{plk}(r_{\rm f},r_{\rm i};t)=\frac{1}{\sqrt{2\pi{\rm i}\varepsilon}}\int_{-\infty}^{\infty}\frac{{\rm d}\lambda_1}{2\pi}\sqrt{2l+1}\sqrt{4\pi}{\rm i}^k\widetilde{J}_k\left(\frac{r_0r_1}{\varepsilon}\right)$$
$$\times j_l\left(\frac{\sqrt{2\pi\omega}\lambda_1r_1}{\sqrt{V}}\right)f_1(\lambda_1)J_p\left(\frac{H}{2c}r_0r_1\right)$$
$$\times\exp\left\{{\rm i}\varepsilon\left[\frac{1}{2\varepsilon^2}(r_1-r_0)^2-V(r_1)\right]\right\}\cdot \quad (34)$$

The exponential represents a one-dimensional problem and in the present case of the one-dimensional hydrogen-like atom the following expression is valid

$$\frac{1}{\sqrt{2\pi{\rm i}\varepsilon}}\exp\left\{{\rm i}\left[\frac{1}{2\varepsilon}(r_n-r_{n-1})^2-\varepsilon V(r_n)\right]\right\}=$$
$$\left\langle r_n|{\rm e}^{-{\rm i}H\varepsilon}|r_{n-1}\right\rangle=\sum_{M=1}^{M_{\max}}R^\star_M(\rho_n)R_M(\rho_{n-1}){\rm e}^{-{\rm i}E_M\varepsilon} \quad (35)$$

where we have set

$$\rho=2Zr/M \quad (36)$$

and we have the expressions

$$E_M=-\frac{Z^2}{2M^2} \quad (37)$$

$$R_M(\rho)=\frac{Z^{1/2}}{M^{3/2}}\rho{\rm e}^{-\rho/2}L^{(1)}_{M-1}(\rho) \quad (38)$$

where $L_n^{(\alpha)}$ represents a generalized Laguerre polynomial.

The wave function of the ground state of the one electron atom is given as

$$|\psi_g(\mathbf{r})\rangle=R_{10}(r)Y_{00}(\hat{\mathbf{r}}) \quad (39)$$

where

$$R_{10}(r)=2Z^{3/2}{\rm e}^{-Zr}. \quad (40)$$

We have assumed to a first approximation that the external fields do not disturb the wave function of the ground state considerably.

Therefore the survival probability amplitude of the ground state is given as

$$A(t)=\frac{1}{\sqrt{\mu\mu'N(t)}}\left\langle R_{10}(r_{\rm f})|\widetilde{K}_{000}(r_{\rm f},r_{\rm i};t)|R_{10}(r_{\rm i})\right\rangle \quad (41)$$

and the survival probability follows.

In Figure 1 we give the survival probability for the ground state of the hydrogen atom as a function of time and for various values of the magnetic field. We observe that large values of the magnetic field assist transition to other states.

Moreover we observe that population trapping in the initial state occurs for various values of the magnetic field.

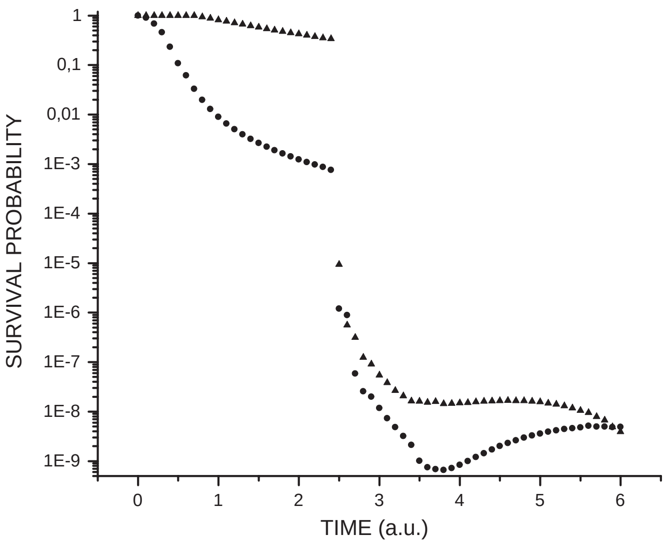

Fig. 1. Survival probability of the ground state of the hydrogen atom in squeezed vacuum and a magnetic field. The following parameters have been set: $\nu = 1.0$, $\kappa = 0.9$ MHz, $\omega = 0.4998$ a.u., $H = 1.0$ a.u. (triangles) and 1000.0 a.u. (circles).

4 Conclusions

At the present paper we investigate the dynamics of a one-electron atom in the presence of both linearly polarized squeezed vacuum and a magnetic field along the photonic field polarization. A degenerate parametric amplifier could generate the squeezed vacuum for instance. We use path integral methods and we treat all the terms on an equal footing the only approximation performed, being the Markovian approximation. We apply our methods to the case of the dynamics of the ground state of a one-electron atom. Our methods are tractable and we believe that they give new aspects on the interaction of radiation with matter in the presence of fields.

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